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Letter to the Editor

Condition of chaotic vibration in a centrifugal governor

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1. Introduction

The centrifugal governor may be the earliest controller used to adjust the speed of rotating machinery. Since the simplicity in its structure, the centrifugal governor is used in turbine engine or other machines until now. Recently, it has been found that chaotic motion may appear when its rotational speed undergoes a harmonic variation which may be caused as the load of the machine changes [1]. In this note, the condition for chaotic vibration of a one-degree-of-freedom simplified governor model is derived with Melnikov function. The results may be useful in design of the centrifugal governor.

2. Equation of motion

The centrifugal governor studied is shown in Fig. 1. When the friction and masses of the collar and of the rods are neglected, the motion of the governor is described by the differential equation

$$mR^2\ddot{\theta} = \frac{mR^2\Omega^2}{2}\sin 2\theta - c\dot{\theta} - mgR\sin\theta. \tag{1}$$

It is assumed that the centrifugal governor rotates at a constant speed W and undergoes a variation with a harmonic term $\varepsilon\cos\omega t$, thus the rotating speed of the governor is expressed as

$$\Omega = W(1 + \varepsilon\cos\omega t), \tag{2}$$

where ε is the disturbance coefficient.

By defining $\theta = x$ and $\dot{x} = y$, Eq. (1) is written as

$$\begin{cases} \dot{x} = y, \\ \dot{y} = \dfrac{W^2}{2}\sin 2x - \delta y - \dfrac{g}{R}\sin x + \varepsilon W^2 \sin 2x\cos\omega t, \end{cases} \tag{3}$$

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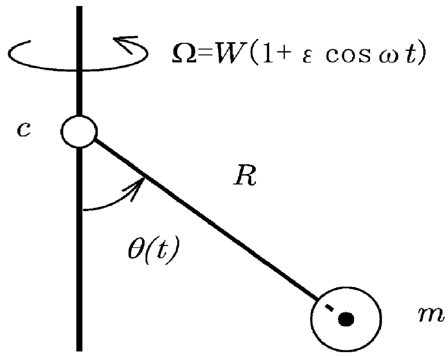

Fig. 1. Model of a centrifugal governor (m: mass of the fly-ball, R: length of the rod, c: constant viscous damping coefficient of the rod bearing).

where $\delta = c/mR^2$. For simplicity, the term ε^2 is ignored and

$$\Omega^2 \simeq W^2(1 + 2\varepsilon \cos \omega t). \tag{4}$$

3. Homoclinic orbit and conditions for chaotic motion

Consider the system

$$\dot{x} = \frac{\partial H}{\partial x}(x, y) + \varepsilon g_1(x, y, \phi, \varepsilon),$$

$$\dot{y} = \frac{\partial H}{\partial y}(x, y) + \varepsilon g_2(x, y, \phi, \varepsilon),$$

$$\dot{\phi} = \omega, \tag{5}$$

where H is Hamiltonian and

$$g_i(x, y, \phi, \varepsilon) = g_i(x, y, \phi + 2\pi, \varepsilon).$$

For the system described in Eq. (5), Melnikov function is expressed as [2]

$$M(W, \omega, \delta, R, \varepsilon) = \int_{-\infty}^{\infty} \left\{ \frac{\partial H(q_0(t, W, R))}{\partial x} g_1\left(q_0(t, W, R), \omega t + \frac{\pi}{2}, \delta, \varepsilon\right) + \frac{\partial H(q_0(t, W, R))}{\partial y} g_2\left(q_0(t, W, R), \omega t + \frac{\pi}{2}, \delta, \varepsilon\right) \right\} \mathrm{d}t. \tag{6}$$

To obtain condition of the parameters of the centrifugal governor for chaotic vibration, the expression of Homoclinic orbit $q_0(t)$ is derived first, then it is taken into the right hand of Eq. (6). Thus the condition will be obtained by $M(W, \omega, \delta, R, \varepsilon) = 0$.

When $\delta = 0$ and $\varepsilon = 0$ in Eq. (3), Taylor's expansion gives

$$\dot{x} = y,$$

$$\dot{y} = \left(W^2 - \frac{g}{R}\right)x - \left(\frac{2W^2}{3} - \frac{g}{6R}\right)x^3 + O(x^4). \tag{7}$$

Then the motion equation of the system is approximated as

$$\ddot{x} = \alpha x - \beta x^3, \tag{8}$$

where

$$\alpha = \frac{RW^2 - g}{R}, \quad \beta = \frac{4RW^2 - g}{6R}. \tag{9}$$

Multiplying both sides of Eq. (8) by $\dot{x}$ and making integration gives

$$\frac{1}{2}(\dot{x})^2 = \frac{\alpha}{2}x^2 - \frac{\beta}{4}x^4 + C, \tag{10}$$

where C is integration constant.

Let $C = 0$, from Eq. (10)

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \pm\sqrt{\alpha x^2 - \frac{\beta}{2}x^4}. \tag{11}$$

The Homoclinic orbit is given by solving Eq. (11):

$$q_0(t, W, R) = \begin{cases} x_0(t) = \pm\sqrt{\dfrac{2\alpha}{\beta}}\,\mathrm{sech}(\sqrt{\alpha}t), \\ y_0(t) = \mp\alpha\sqrt{\dfrac{2}{\beta}}\,\mathrm{sech}(\sqrt{\alpha t})\tanh(\sqrt{\alpha}t). \end{cases} \tag{12}$$

Comparing Eq. (3) with Eq. (5), one obtains

$$\frac{\partial H}{\partial x}(x, y) = y, \quad g_1(x, y, \phi, \varepsilon) = 0,$$

$$\frac{\partial H}{\partial y}(x, y) = \frac{W^2}{2}\sin 2x - \frac{g}{R}\sin x, \quad g_2(x, y, \phi, \varepsilon) = \varepsilon W^2 \sin 2x \cos \omega t - \delta y.$$

Then the Melnikov function expressed in Eq. (6) becomes

$$
\begin{aligned}
M(t_0,\phi_0) = &\int_{-\infty}^{\infty}\left(-\frac{2\varepsilon\delta\alpha^2}{\beta}\,\mathrm{sech}^2(\sqrt{\alpha t})\tanh^2(\sqrt{\alpha t})\right)\mathrm{d}t\\
&-\int_{-\infty}^{\infty}\left[\sqrt{\frac{2\alpha^2}{\beta}}\,W^2\mathrm{sech}(\sqrt{\alpha t})\tanh(\sqrt{\alpha t})\sin\left(2\sqrt{\frac{2\alpha}{\beta}}\,\mathrm{sech}(\sqrt{\alpha t})\right)\right.\\
&\qquad\left.\cos\omega t\cos(\omega t_0+\phi_0)\right]\mathrm{d}t\\
&+\int_{-\infty}^{\infty}\left[\sqrt{\frac{2\alpha^2}{\beta}}\,W^2\mathrm{sech}(\sqrt{\alpha t})\tanh(\sqrt{\alpha t})\sin\left(2\sqrt{\frac{2\alpha}{\beta}}\,\mathrm{sech}(\sqrt{\alpha t})\right)\right.\\
&\qquad\left.\sin\omega t\sin(\omega t_0+\phi_0)\right]\mathrm{d}t\\
\cong &-\frac{4\varepsilon\delta\alpha\sqrt{\alpha}}{3\beta}+\frac{2\pi}{\beta}AW^2\omega^2\exp\left(-\frac{\pi B\omega}{2\sqrt{\alpha}}\right)\sin(\omega t_0+\phi_0),
\end{aligned}
\tag{13}
$$

where $A = 1.15004575 \pm 0.00746936$ and $B = 1.21715509 \pm 0.00288709$ which are generated in the integration.

Hence, letting $\sin(\omega t_0 + \phi_0) = 1$ and $M(t_0, \phi_0) = 0$, yields the condition for chaotic vibration of the system

$$
\delta \leqslant \frac{3\pi\varepsilon A W^2\omega^2}{\sqrt{(W^2 - g/R)^3}}\exp\left(-\frac{\pi B\omega}{2\sqrt{W^2 - g/R}}\right). \tag{14}
$$

Fig. 2 shows the critical damping values when $\varepsilon = 0.5$, $g = 9.81\ \mathrm{m/s^2}$, $R = 0.85$ m, $1 \leqslant \omega \leqslant 50$ (rad/s) and $4 \leqslant W \leqslant 50$ (rad/s) for example. If the value of the damping coefficient δ is beneath the surface, the response of the centrifugal governor could be chaotic. One of the results

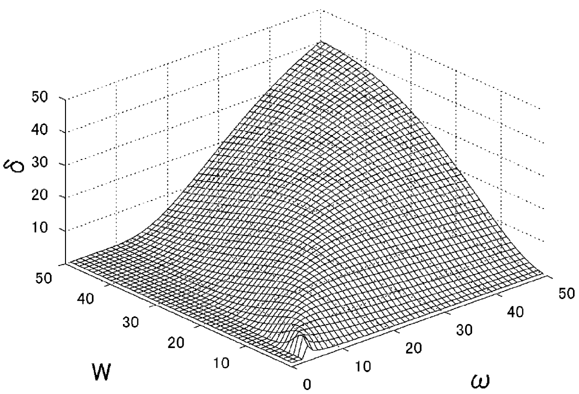

Fig. 2. Critical surface of damping coefficient δ for chaotic responses.

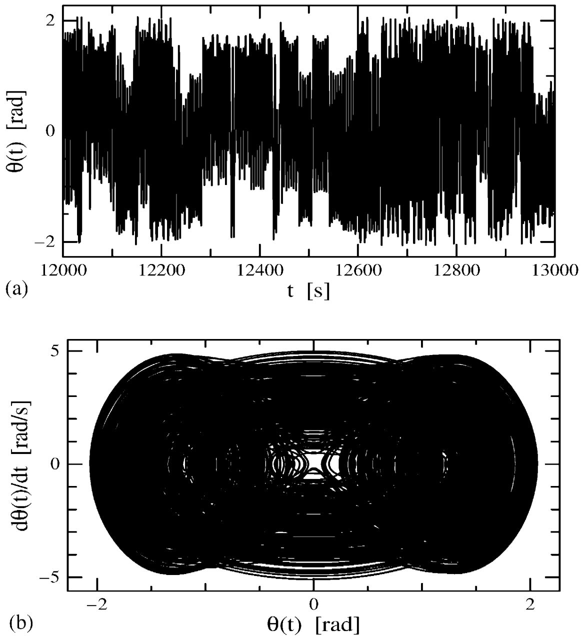

Fig. 3. Chaotic vibration of the fly-ball governor ($\varepsilon = 0.5$, $g = 9.81\ \mathrm{m/s^2}$, $R = 0.85$ m, $\omega = 1.32$ rad/s, $W = 4.4$ rad/s, $\delta = 0.001$, initial condition: $\theta_0 = 0.01$ rad, $\dot{\theta}_0 = 0$ rad/s): (a) time history of $\theta(t)$, (b) the corresponding phase-plane motion.

of numerical simulation is shown in Fig. 3. The damping coefficient is $\delta = 0.001$ which is under the surface in Fig. 2. Since the computed dominant Lyapunov exponent [3,4] of the time history in Fig. 3a is 2.29, the response of the system is chaotic.

4. Concluding remarks

In this shot note, the condition for chaotic response in a centrifugal governor was derived analytically by applying Melnikov function. The simplification was made in order to obtain Eq. (8) which is used in computation of the Homoclinic orbit. Thus there exists error in the derived condition. However, the analytical condition is still useful since it gives a guideline for parameter selection in design of the governor. If more accurate expression of the condition is needed, the same procedure can be used with higher order approximation of the motion equation.

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